

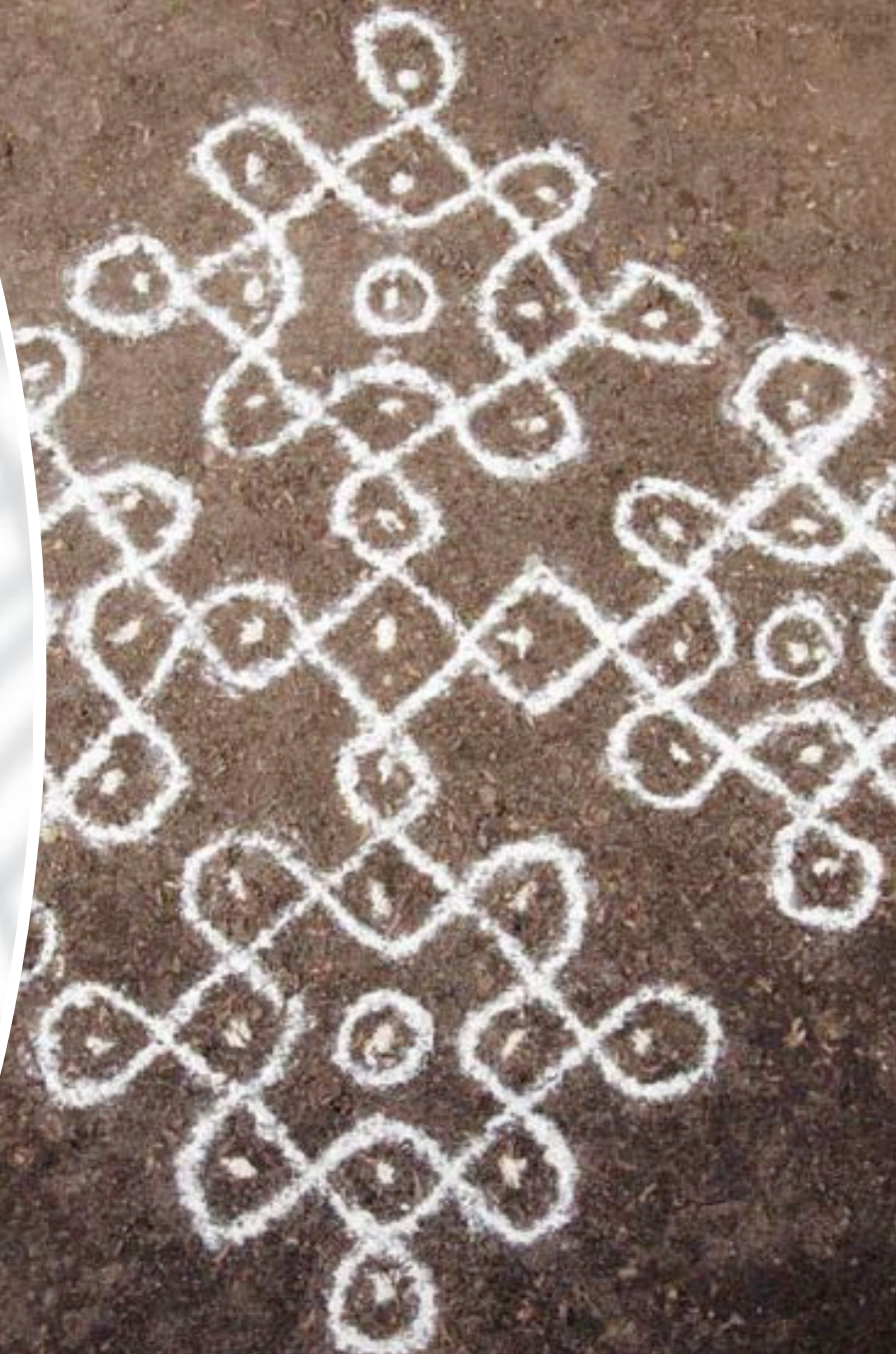
Figures on the Threshold

A Classroom Capsule from my *Ethnomathematics* capstone seminar course, linking

- ❖ Formal Language Theory
- ❖ Traditional Art of *Kolam* Drawing
- ❖ Generative AI



Marcia Ascher



Formal Language Theory is a branch of theoretical computer science that deals with the study of formal languages, which are sets of strings with specific rules governing their formation. It provides a foundation for understanding and generating languages, which is crucial in various areas of computer science and artificial intelligence.

Alphabet : A fixed set of symbols for the language

Axiom: A starting string of symbols

Rewriting Rules: A set of rules for creating new strings of symbols from previous ones

Outcomes: The strings of symbols that result from applying the rewriting rules

Example:

Alphabet: A, B, C

Axiom: ABB

Rewriting Rules: $A \rightarrow BC$

$B \rightarrow A$

$C \rightarrow C$

Start: ABB

Outcome 1: (BC)AA

Outcome 2: AC(BC)(BC)

Outcome 3: (BC)CACAC

etc

Moving from a String of Symbols to Picture Languages

Example:

Alphabet: F, +, -

Axiom: F - F

Rewriting Rules: F \rightarrow F + F

+ \rightarrow +

- \rightarrow -

Start: F - F

Outcome 1: [F + F] - [F + F]

Outcome 2: [(F+F) + (F+F)] - [(F+F) + (F+F)]

Outcome 3: F+F+F+F+F+F+F+F-F-F+F+F+F+F+F+F+F+F

etc

Computer Commands (Turtle Moves)

F: move forward by step length s while drawing a line

f: move forward by step length s without drawing a line

+: turn left (counterclockwise) through an angle of d degrees

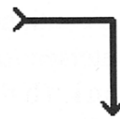
- : turn right (clockwise) through an angle of d degrees

one unit

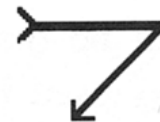
90°

Turtle interpretation of

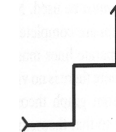
Start: F - F



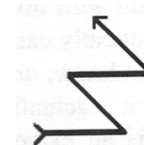
If $d = 135^\circ$, Start: F - F



Outcome 1: [F + F] - [F + F]

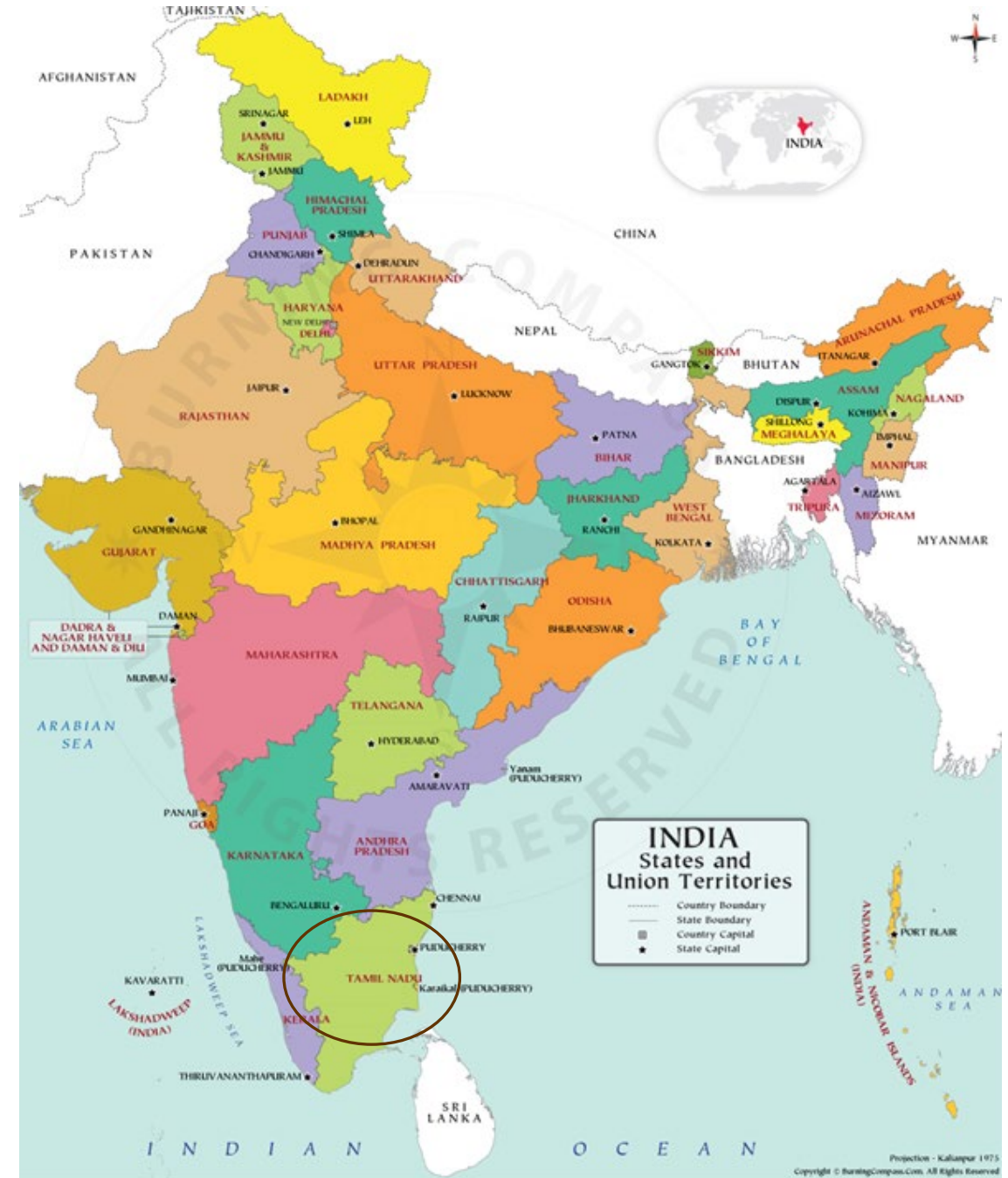


Outcome 1: [F + F] - [F + F]



The *Kolam*

- *Kolam* is the traditional art form of the people of Tamil Nadu in the southeastern region of India
- It is a design placed each morning by women on the thresholds of their homes, created by using rice flour held in the hand and slowly trickled in a thin stream from between the index and the middle finger
- Philosophically, the threshold is a boundary, but a permeable one, symbolic of the boundary between the inner world of the mind and emotion and the outer world of the landscape and action. It is also identified with transition points in the cosmic cycle, related both to the lives of individuals, and to the seasonal round
- Thus, beyond their aesthetic value, the threshold decorations are chosen to appropriately mark life-passage events, rituals, or as the prelude to worship of deities.



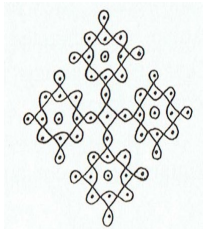
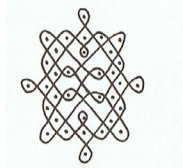
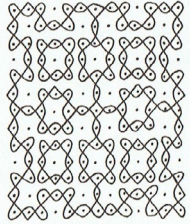
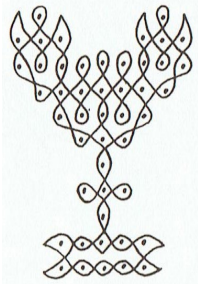
The Art of *Kolam* Drawing

Kolam figures vary widely in style. Many but not all begin with a grid of dots (*pulli*). The configuration of the grid is an important guide to placing the rest of the figure

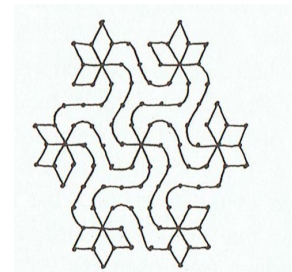
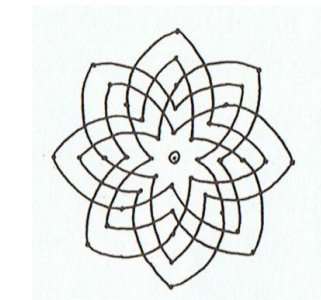
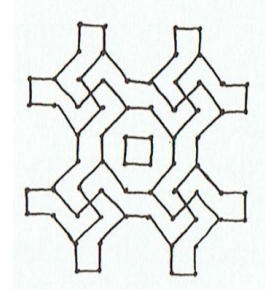
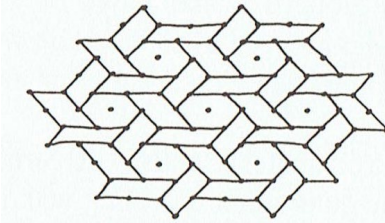
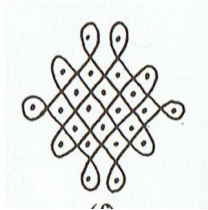
Some kolam are constructed by drawing lines connecting the pulli;
for others, the lines go around the pulli



Kolams with lines connecting the pulli



Kolams with lines around the pulli



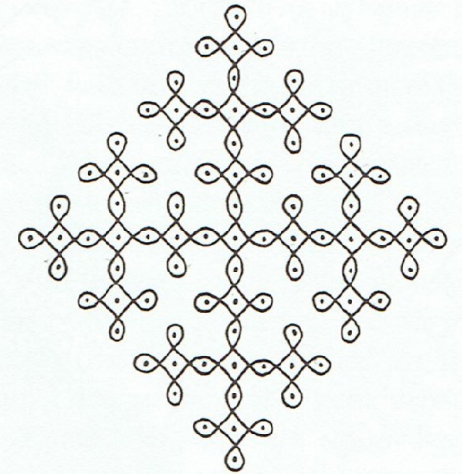
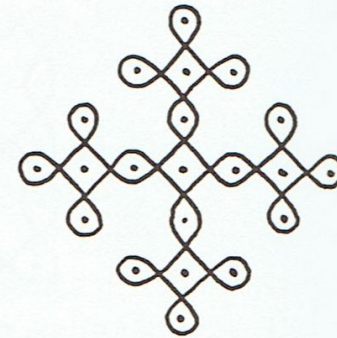
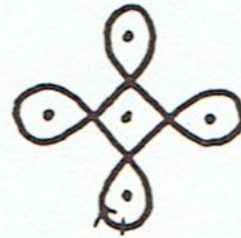
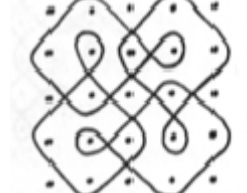
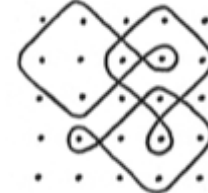
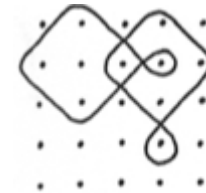
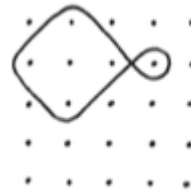
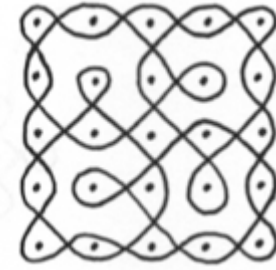
Kolam Drawing and its Linkage to Picture Languages

A group of computer scientists of Madras Christian College in Tamil Nadu created the bridge between kolam and computer science by using picture languages to generate them

They studied the steps used by the women to create certain designs and noticed the following –

- The women proceeded to draw a closed curve and then repeated the same curve a few times, but each time they rotated the curve to complete the design
- Larger curves are made up of several copies of the smaller curves with patterned repetitions

They sought a picture language whose outcomes would be the different combination of basic units produced through specific, formal rules for combining the units



Kolam Moves

The Madras Christian College computer scientists defined seven *kolam moves* based on Tamil women's descriptions of their actions:

F: move forward one unit while drawing a line

R₁: move forward while making a half-turn right

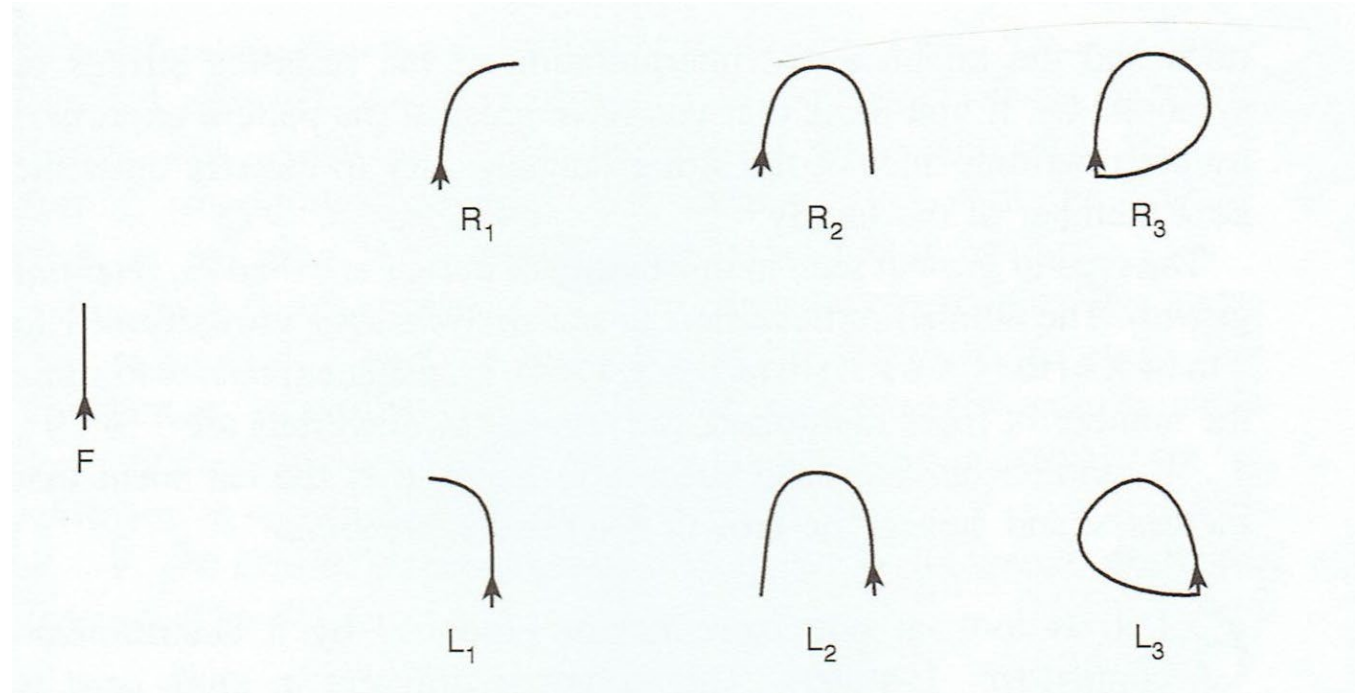
R₂: move forward while making a u-turn to the right

R₃: move forward while making a full loop to the right

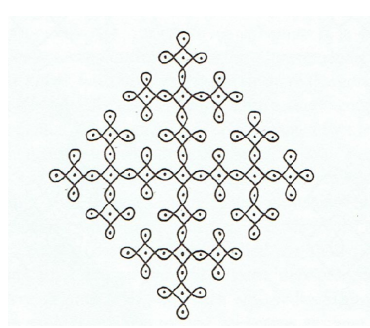
L₁: move forward while making a half-turn left

L₂: move forward while making a u-turn to the left

L₃: move forward while making a full loop to the left



A Language that produces



Alphabet: F, R_1, R_3

Axiom: $R_1FR_3FR_3FR_3FR_1$

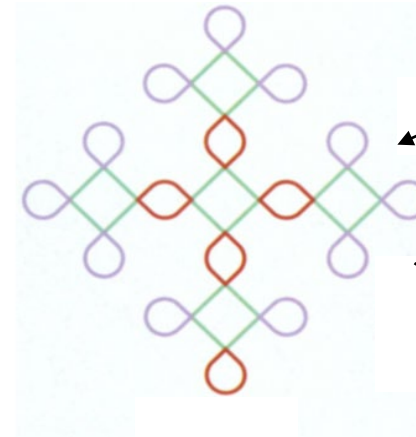
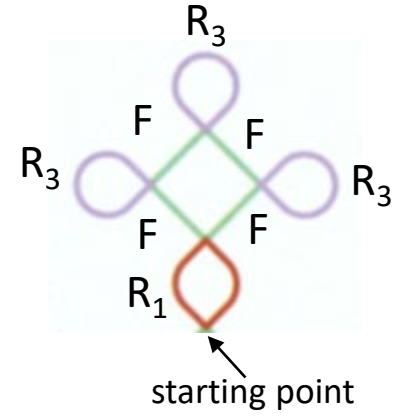
Rewriting Rules: $F \longrightarrow F$

$R_1 \longrightarrow R_1FR_3FR_1$

$R_3 \longrightarrow R_1FR_3FR_3FR_3FR_1$

Outcome 1: $(R_1FR_3FR_1)F(R_1FR_3FR_3FR_3FR_1)$
 $F(R_1FR_3FR_3FR_3FR_1)F(R_1FR_3FR_3FR_3FR_1)F(R_1FR_3FR_1)$

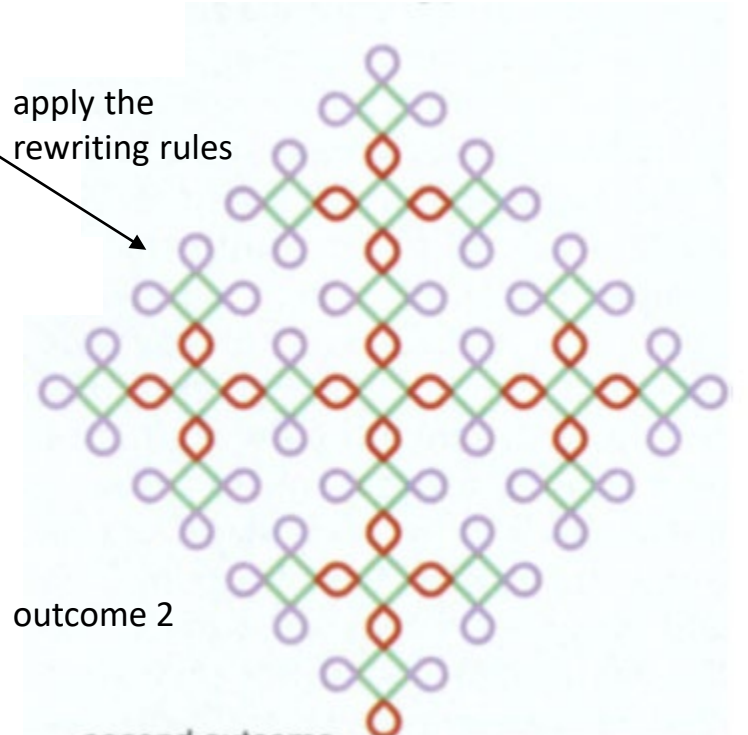
Pictorial Representation of the Axiom



outcome 1

apply the rewriting rules

apply the rewriting rules



outcome 2

Generative AI and its Linkage to formal languages and rewriting rules

I asked ChatGPT...



how is generative AI linked to formal languages and rewriting rules



Generative AI is linked to formal languages and rewriting rules through the use of computational models and algorithms inspired by formal language theory. The connection lies in how generative AI systems generate new data, such as text, images, or other types of content.

Formal languages are a theoretical framework used in computer science and linguistics to describe sets of strings with specific patterns and structures. They are defined by a set of symbols and rules for combining those symbols to form valid strings. These rules are typically expressed as production rules or rewriting rules.

In generative AI, formal languages and rewriting rules play a crucial role in creating and generating content. Here's how they are connected:

Generative AI and its Linkage to formal languages and rewriting rules

I asked ChatGPT...

1. **Generative Models:** Generative AI models are designed to learn and model the underlying distribution of a dataset, enabling them to generate new samples that resemble the training data. These models can be based on a variety of techniques, such as probabilistic models, autoregressive models, and more.
2. **Language Models:** Language models, a type of generative AI model, are specifically designed to generate human-like text. They learn the statistical patterns and relationships between words in a language corpus, allowing them to produce coherent and contextually relevant sentences.
3. **Rewriting Rules in Language Generation:** Language generation in AI often involves applying rewriting rules to existing sentences to create new ones. These rules dictate how certain patterns or phrases can be transformed into other patterns, preserving grammatical correctness and semantic coherence. Such rewriting rules can be handcrafted or learned from data.
4. **Context-Free Grammars:** Context-free grammars are a type of formal language that defines a set of rules for generating strings by replacing non-terminal symbols with sequences of terminal symbols. These grammars are often used to model the syntactic structure of languages and are used in various generative AI techniques.
5. **Transformers and Sequence-to-Sequence Models:** Transformer-based architectures, like the GPT (Generative Pre-trained Transformer) models, are commonly used in language generation tasks. These models employ self-attention mechanisms, enabling them to capture long-range dependencies and generate coherent sentences based on the input context.
6. **Neural Machine Translation (NMT):** Neural Machine Translation is a popular application of generative AI that uses sequence-to-sequence models to translate text between different languages. In NMT, the translation process can be viewed as a series of rewriting rules applied to a source sentence to generate the target translation.

🔄 Regenerate

Works Cited

1. Ascher, M. (2002). *Mathematics elsewhere: An exploration of ideas across cultures*. Princeton University Press.
2. Ascher, M. (2002). The Kolam Tradition: A tradition of figure-drawing in southern India expresses mathematical ideas and has attracted the attention of computer science. *American Scientist*, 90(1), 56-63.